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## Abstract

A physical phenomenon is found through the analysis of the Hodgkin-Huxley's model, that is according to the Maxwell field equations an exciting neuron can yield magnetic field. The magnetic signals are an important output of the neuron as some type of information. There exists an electromagnetic field in the brain. The brain is not a net, but is a field. According to this idea a proper physical explanation has been made for the Hebbian Hypothesis, and a new neuron model has been proposed. A mathematical Memory-Learning Relation has been derived from the new neuron equations. Finally, a new theory, i.e. Neural Electromagnetic Field (NEF) theory is advanced.

## 1 Introduction

The recent achievements of the neural network research, compared with those obtained in the 1960's are only amendment and no new principles or theories have been advanced. For the development of neuro-information science it is necessary to put forward new principles and new theories.

## 2 Two Types Output of an Exciting Neuron

### 2.1 Hodgkin-Huxleys Model

In the paper [1], Hodgkin-Huxley suggested that the electric behaviors of the membrane of a neuron may be represented by the network shown in Fig. 1. The current can be carried through the membrane either by charging the membrane capacity or by movement of ions through the resistances in parallel with the capacity.

### 2.2 An Exciting Neuron Can Yield Magnetic Field

Hodgkin-Huxleys electric network can be made a equal change shown in Fig. 2. In this new network we let  $J$  be an ionic cur-

rent,  $j$  be a capacity current or displacement current,  $D$  be electric flux density of the capacity. According to Maxwell field equations the electric circuit can yield a magnetic field, and the magnetic field strength will be  $H$ :

$$\text{rot} \mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t} \quad (1)$$

From the analysis above we can conclude that when a neuron is exciting, a magnetic field will be yielded. In other words an exciting neuron can yield two types of output, i.e. one type is electric signals, another type is magnetic signals. There exists an electromagnetic field in the brain. The brain is not a net, but is a field.

## 3 Explanation of Hebbian Learning Hypothesis

A physical explanation of Hebbian learning hypothesis can be made in a three dimensional field according to the electromagnetic induction rule. Now we can regard Hebbian learning hypothesis as Hebbian Rule. In one way we may understand why the Network theory can not properly explain the Hebbian learning hypothesis.

## 4 New Neuron Model

First, based on McCulloch-Pitts neuron model we describe the main features analyzed above as a conceptual model shown in Fig. 3. Second, by using Maxwell field equations and Faraday's rule we put the concept model into mathematical form. The mathematical neuron model proposed as follows:

$$\tau_u \frac{du(t)}{dt} = -u(t) + \sum_{i=1}^n w_i^D p_i - \mu \sum_{j=1}^m w_j^H \frac{dq_j}{dt} - \mu \frac{dv(t)}{dt} - h \quad (2)$$

$$\tau_v \frac{dv(t)}{dt} = -v(t) + \epsilon \sum_{i=1}^n w_i^D p_i + \sum_{j=1}^m w_j^H q_j + \epsilon u(t) \quad (3)$$

$$O_u = f[u(t)] \quad (4)$$

$$O_v = f[v(t)] \quad (5)$$

where

$p_1, p_2, \dots, p_n$  are electric input signals from  $n$  exciting neurons;

$q_1, q_2, \dots, q_m$  are magnetic input signals from  $m$  exciting neurons;

$u(t)$  is variation of the membranous electric potential in terms of time;

$v(t)$  is variation of the membranous magnetic potential in terms of time;

$O_u$  is electric output signals of the neuron;

$O_v$  is magnetic output signals of the neuron;  
 $h$  is threshold value;  
 $w_i^D$  is synaptic efficiency or synaptic weight from  $i$  neuron;  
 $w_j^H$  is hebbian efficiency or learning weight from  $j$  neuron;  
 $\mu$  is magnetoelectric coefficient;  
 $\epsilon$  is electromagnetic coefficient;  
 $\tau_u$  is constant in terms of time;  
 $\tau_v$  is constant in terms of time;  
 $t$  is time.

This new neuron model is an information model and implies the relation between memory and learning. In fact the model is a three dimensional model.  $u(t)$  and  $v(t)$  are potential functions in a three dimensional field. The neuron equations also can be represented as:

$$\begin{aligned} \tau_u \frac{\partial u(x, y, z, t)}{\partial t} = & -u(x, y, z, t) \\ & + \sum_{i=1}^n w_i^D p_i(x, y, z, t) \\ & - \mu \sum_{j=1}^m w_j^H \frac{\partial q_j(x, y, z, t)}{\partial t} \\ & - \mu \frac{\partial v(x, y, z, t)}{\partial t} - h \end{aligned} \quad (6)$$

$$\begin{aligned} \tau_v \frac{\partial v(x, y, z, t)}{\partial t} = & -v(x, y, z, t) \\ & + \epsilon \sum_{i=1}^n w_i^D p_i(x, y, z, t) \\ & + \sum_{j=1}^m w_j^H q_j(x, y, z, t) \\ & + \epsilon u(x, y, z, t) \end{aligned} \quad (7)$$

The neuron equations can be regarded as the basis of Neural Electromagnetic Field Theory.

## 5 Memory-Learning Relation (MLR)

We can convert  $\sum_{i=1}^n w_i^D p_i$  and  $\sum_{j=1}^m w_j^H q_j$  in the neuron equations into vector form. Thus, the neuron equations will be

$$\tau_u \frac{du}{dt} = -u + \mathbf{W}^D \mathbf{P} - \mu \mathbf{W}^H \frac{d\mathbf{Q}}{dt} - \mu \frac{dv}{dt} - h \quad (8)$$

$$\tau_v \frac{dv}{dt} = -v + \epsilon \mathbf{W}^D \mathbf{P} + \mathbf{W}^H \mathbf{Q} + \epsilon u \quad (9)$$

where

$$\begin{aligned} \mathbf{W}^D &= [w_1^D, w_2^D, \dots, w_n^D]; \\ \mathbf{W}^H &= [w_1^H, w_2^H, \dots, w_m^H]; \\ \mathbf{P} &= [p_1, p_2, \dots, p_n]^T; \\ \mathbf{Q} &= [q_1, q_2, \dots, q_m]^T. \end{aligned}$$

The Memory-Learning Relation (MLR) can be derived from the neuron equations, and the MLR is

$$\alpha \mathbf{P} \frac{d\mathbf{W}^D}{dt} = -\mu \frac{dv}{dt} + \mu \mathbf{Q} \frac{d\mathbf{W}^H}{dt} \quad (10)$$

This equation (MLR) can account for the mechanism of memory and learning with mathematical principles. In the MLR  $\alpha$  is memory factor,  $\mu$  is learning factor. From the MLR we can conclude that the relation between memory and learning is expressed through the neurons magnetic field and a neuron has learning abilities. The MLR can be called Learning Rule in the NEF theory.

## 6 Conclusion

This paper presents a theory of Neural Electromagnetic Field for the study of neuro-information science. The Neuron Equations construct the fundamentals of the NEF theory, and the MLR is a development of the NEF theory.

## References

- [1] A.L.Hodgkin and A.F.Huxley, *A Quantitative Description of Membrane Current and Its Application to Conduction and Excitation in Nerve*, J.Physiol., 117, 500-544, 1952.
- [2] W.S. McCulloch and W.H. Pitts, *A Logical Calculus of the Ideas Immanent in Nervous Activity*, Bull.Math.Biophys., 5, 115-133.

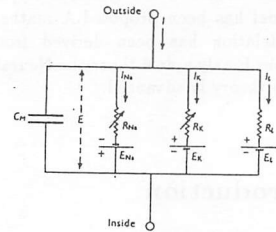


Figure 0.1: H-H's network of a neuron membrane

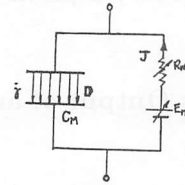


Figure 0.2: An equal circuit of H-H's network

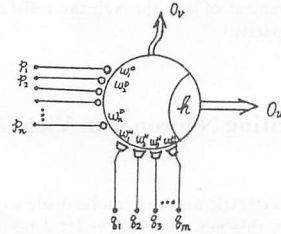


Figure 0.3: Neuron concept model